

# Mean Streamwise Spacing of Organized Structures in Transitional and Developed Bounded Turbulent Flows

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The mean periods of passage of organized structures detected by a wide variety of criteria in all regions of turbulent boundary layers, pipes, and channel flows over a range of two Reynolds number decades are compared to that of the transition "spike." When scaled with the local mean velocities and the shear layer thicknesses to form a nondimensional streamwise spacing, they fall within a fairly narrow range of  $1.5 \leq \bar{T}U/\delta \leq 3.5$ . The streamwise spacing shows little variation across most of the shear layer for any particular data set.

THE quantitative similarity between some of the characteristic features of the transition to turbulence in bounded shear flows and the identifiable features of organized structures in these flows after they have developed has tempted many researchers to seek a relationship between them. Specifically, the existence of unstable local shear layers and counterrotating pairs of vortices in both phenomena seem to be very suggestive.<sup>1,4</sup> Coles and Barker<sup>5</sup> have synthesized a turbulent boundary layer with all the usual average characteristics by creating a regular array of turbulent "spots" that merge as they go downstream but remain identifiable. They suggest that the "spot" may be the basic "building block" of the turbulent layer. Zilberman et al.<sup>6</sup> were able to track a turbulent "spot" for 70 average turbulent boundary-layer thicknesses as it convected downstream and interacted with its turbulent surroundings. Recently Perry et al.<sup>7</sup> have visually shown that the average "spot" structure can be explained in terms of three-dimensional disturbances of the cross-stream vortex filaments that evolve into an array of "hairpin"-like vortices. Leonard<sup>8</sup> obtains essentially the same result by numerically following a local region of perturbed cross-stream vortex filaments in a Blasius boundary layer that evolves into a structure with features similar to a turbulent "spot." In the developed boundary layer, Head and Bandyopadhyay<sup>9</sup> have visually observed "hairpin" vortices that extend from near the wall all the way across the layer for a range of Reynolds numbers of  $600 \leq Re_\theta \leq 9400$ , where  $\theta$  is the momentum thickness. These vortices appear to be the basic organized structure of the boundary layer.

Some 20 years ago Klebanoff et al.<sup>1</sup> carried out a detailed examination of the breakdown of two-dimensional waves in a laminar boundary layer generated by small-amplitude vibrations of a ribbon and that amplify in agreement with linear stability analysis. These waves develop three-dimensionality as they grow downstream and then abruptly break down into high-frequency turbulent fluctuations. The onset of breakdown is characterized by the appearance of a "spike" in the periodic velocity field. By examining the spatial and temporal characteristics of the region around the occurrence of the "spike," they concluded that it is the manifestation of a stretching "hairpin" eddy. They found the mean convection velocity of the eddies to be  $0.64U_\infty$  and the mean period of passage to be about  $2.6\delta/U_\infty$  in a stationary frame of reference.

As noted and commented on earlier by Frenkiel and Klebanoff,<sup>10</sup> these values are strikingly similar to values for the mean period between passages and the mean convection velocity of organized structures observed in many experiments that examined developed bounded shear flows over the last 20 years. This has led us to gather all of the existing data we could find, with the necessary parameters values provided, to make a more exact and comprehensive comparison. This may be considered an extension of an earlier data gathering effort by Laufer and Badri Narayanan.<sup>11</sup>

The length scale used to normalize the mean period is the thickness of the shear layers for each experiment  $\delta$ , i.e., the average boundary-layer thickness, the pipe radius, or the half channel width. Initially we used the freestream velocity of the boundary layer  $U_\infty$  and the centerline velocities of the pipe and channel as the normalizing velocity scale. This was done because it has long been stated that the mean period scales with the outer flow variables.<sup>11</sup> Indeed, a constant value of  $\bar{T}U_\infty/\delta = 5$  has been widely accepted, although values of 2.5-10 have been reported. Additionally, Klebanoff et al.<sup>1</sup> found the relatively constant value of  $2.6\delta/U_\infty$  for their transition measurements at  $0.3 \leq y/\delta \leq 0.6$ , as mentioned above. Although, as seen in Fig. 1, this normalization collapsed the data fairly well for  $y/\delta \geq 0.075$ , there is a very large spread,  $0.70 \leq \bar{T}U_\infty/\delta \leq 10$ , for the "burst" structures detected near the boundaries. Bandyopadhyay<sup>12</sup> has recently made clear that, while  $\bar{T}U_\infty/\delta$  should be independent of  $Re_\theta$ , it will still depend on the flow shape factor as well as on another unknown constant for any particular flow that possibly is affected by the pressure gradient. Thus  $\bar{T}U_\infty/\delta$  should not be, even in principle, a universal constant, although flows with zero or small pressure gradients should exhibit a relatively narrow range of values. Viscous scaling with the friction velocity and the kinematic viscosity, the obvious alternative, did not collapse the data at all.

We therefore decided to normalize  $\bar{T}$  with the mean velocity at the location where the structures were detected to obtain a mean streamwise structure spacing. This can be justified on physical grounds with the following argument. If the structures being detected by all these methods are recurring features or events generated by "hairpin" vortices that are lifted and stretched by the shear flow through the boundary layer and that are distributed with a mean streamwise spacing in their flow region of origin near the wall, then the mean distance between them should be roughly constant across the layer, although their mean local convection velocity and thus their mean period will vary. If the mean local convection velocities were accurately known, multiplying the mean period by these values should yield a nearly constant structure spacing across the entire shear layer. Lacking accurate knowledge of the convection velocities for most of the experiments, the most reasonable alternative appears to be the

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Fig. 1  $\bar{T}U_\infty/\delta$  as a function of  $y/\delta$  ( $i$  denotes intermittency data).

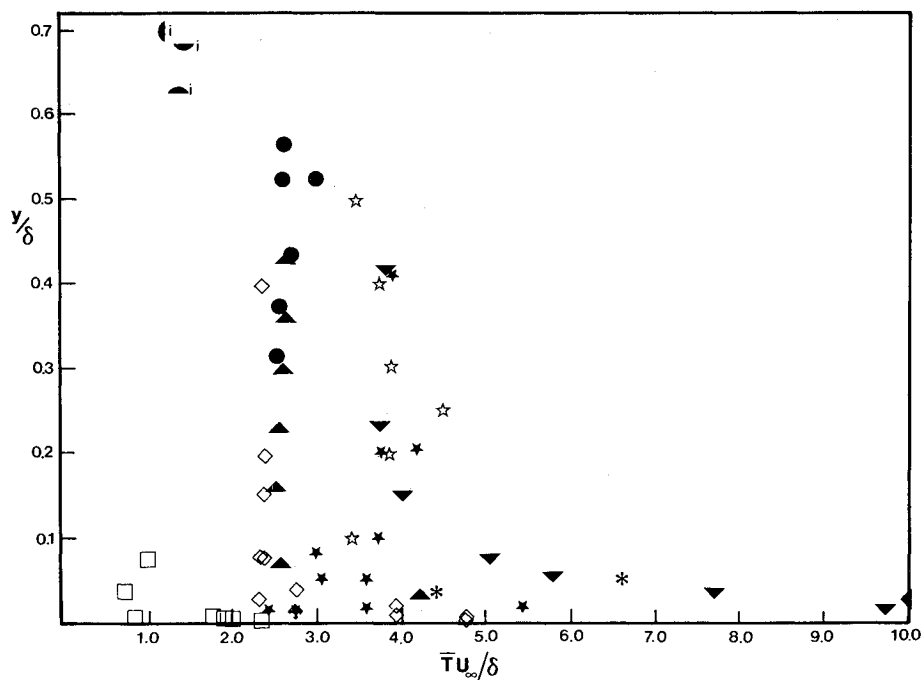
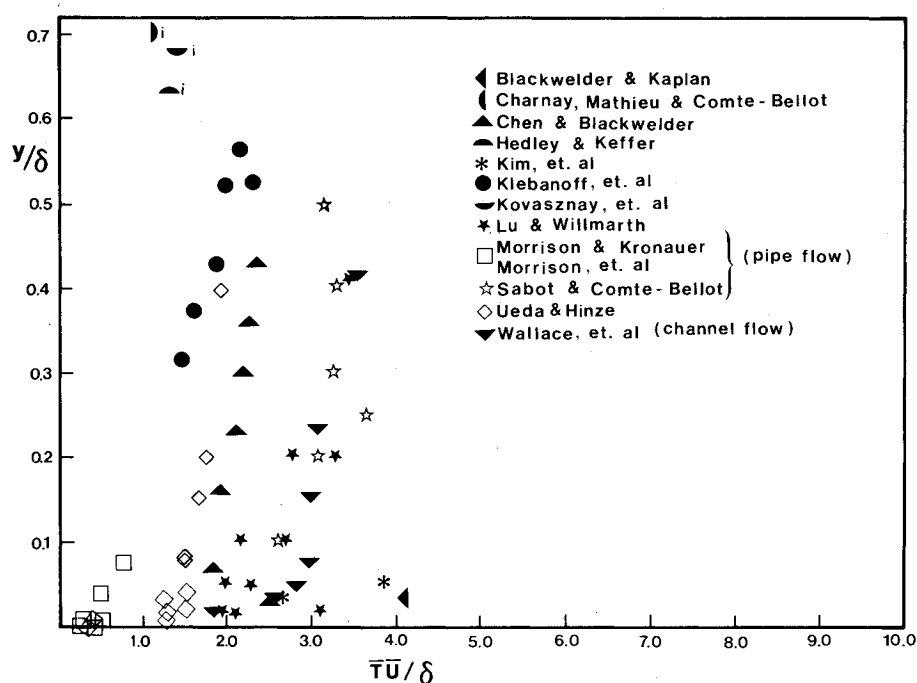


Fig. 2  $\bar{T}U/\delta$  as a function of  $y/\delta$  ( $i$  denotes intermittency data).



local mean velocity. Indeed, this provided a much better collapse of the data viewed as a whole, as seen in Fig. 2. The slight trend of most of the data sets toward larger values of  $\bar{T}U/\delta$  with increasing  $y/\delta$  seems to indicate that the structures convect with velocities somewhat greater than the local mean near the wall and somewhat less than the local mean at larger  $y/\delta$ . This is in fact reported by many of the authors and agrees with earlier space-time correlations.<sup>13</sup> It may also reflect the likelihood that not all the vortices are able to migrate across the entire shear layer.

In the region  $y/\delta \leq 0.4$  (with intermittency,  $\gamma = 0.0$  for boundary layers and where structures from the other walls are negligible for pipes and channels) organized structures have been detected across the layers using power spectral methods,<sup>14,15</sup> quadrant-amplitude analysis of intense Reynolds stress contributions of "bursts" and "sweeps,"<sup>16,17</sup>

pattern recognition of large streamwise straining,<sup>18</sup> detection of periods of high streamwise turbulent kinetic energy,<sup>19</sup> detection of internal temperature fronts using heat as a passive containment,<sup>20</sup> and detection of streamwise straining by bandpass filtering.<sup>21</sup> For  $y/\delta \geq 0.1$  the experiments yield  $1.5 \leq \bar{T}U/\delta \leq 3.5$ . This range of values is remarkably small considering the quite different detection methods used with all of their attendant uncertainties. For  $y/\delta \leq 0.1$  the data spread is somewhat larger, but this is primarily due to the low values obtained from the power spectra<sup>14,15</sup> where, in fact, the convection velocity was reported to be considerably larger than the local mean for all the data shown and to the viscous sublayer results of Ueda and Hines<sup>21</sup> where "bursting" is unlikely to occur. The values for the transition experiment of Klebanoff et al.<sup>1</sup> fall in the range  $1.4 \leq \bar{T}U/\delta \leq 2.25$ , i.e., in about the same range as the developed flow experiments.

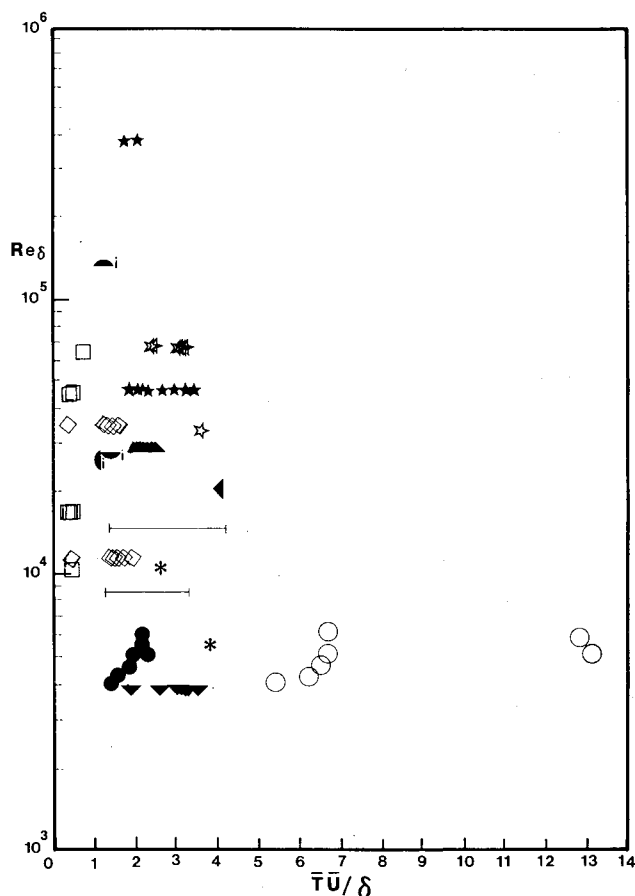


Fig. 3  $\bar{T}U/\delta$  as a function of  $Re_\delta$ . ( $\circ$  denotes the primary waves of Klebanoff et al. and  $|$  values obtained from the stability diagram.)

More important, however, than the relatively small range of  $\bar{T}U/\delta$ , which should show some variation between experiments according to Bandyopadhyay,<sup>12</sup> is the small variation of  $\bar{T}U/\delta$  with  $y/\delta$  for any particular data set. This implies a relatively constant spacing between structures extending across most of the boundary layer.

Also shown in Fig. 2 are the mean intermittency periods of the intermittent turbulent "bulges" in the outer region of boundary layers<sup>22-24</sup> where  $\gamma=0.7$ , which is roughly the center of the large rotating "bulge" structure.<sup>25</sup> These values lie in the range  $1.1 \leq \bar{T}U/\delta \leq 1.5$ . This intermittency period is a strong function of  $\gamma$  (or  $y/\delta$ ) in the outer ( $y/\delta \geq 1.0$ ) and the inner ( $y/\delta \leq 0.5$ ) regions of the intermittent flow, but for  $0.5 \leq y/\delta \leq 1.0$  the mean period of "bulges" roughly falls in the range found for the fully turbulent region and for the transition "spikes."

Figure 3 shows the same data plotted as a function of  $Re_\delta = U_\infty \delta / \nu$ . It is seen that over about two decades,  $4 \times 10^3 \leq Re_\delta \leq 4 \times 10^5$ , the results are fairly independent of flow Reynolds number as was suggested by Bandyopadhyay.<sup>12</sup> Also shown in this figure are the values of the normalized periods of the primary waves,  $5.4 \leq \bar{T}U_p/\delta \leq 13.1$ , which produced the transition "spikes" in the experiment of Klebanoff et al.<sup>1</sup> Their known phase velocities  $U_p$  have been used in the normalization. These values, which lie along both branches of the linear stability diagram, are much larger than those of the subsequent quasiperiodic ordered motions seen in the flow. There are, however, waves within the linearly unstable region with wavelengths and phase velocities in the range of the turbulence data in Figs. 1 and 2. Specifically, it is interesting to note that the values of  $\bar{T}U_p/\delta$  obtained from the neutral growth curve on a stability diagram for a Blasius boundary layer at and slightly above the Reynolds number

where transition is generally observed to occur fall within the range of the data.

The data gathered above cannot prove that organized structures in the developed regions of bounded turbulent shear flows are generated by similar structures that originate in transition. However, the fact that both cases exhibit structures with mean streamwise spacings that vary only slightly across the flow and that are about the same fraction of the shear layer thickness, when added to their qualitative kinematic similarities, does strongly suggest a relationship. Investigating this remains one of the outstanding problems in turbulence shear flow research.

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